Multiple Duals in Soft Erosion in Multi Scale Soft Morphological Environment

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Abstract- *In this paper, Multiple duals of soft erosion are discussed in multi scale environment. In mathematical morphological environment, erosion and dilation are dual to each other; open and close are dual to each other. But in soft mathematical morphology, the duality is not discussed thoroughly. It is going to be discussed in this paper in detail*

Keywords:. Multiple dual, Duality, Mathematical morphology, Mathematical soft morphology, Soft morphology, Erosion, Dilation, Soft erosion, Soft dilation, Primitive morphological operation.

The paper is divided in to 8 sections.

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I. INTRODUCTION

The human beings have the desire of recording incidents, through images. It has started from early cavemen also. Later, so many techniques, to get the images and so many techniques, to process the images are developed. After assembling of computers, image processing was expanded.

In 1964 G. Matheron was asked to investigate the relationships between the geometry of porous media and their permeability. At the same time, J. Serra was asked to quantify the petrography of iron ores, in order to predict their milling properties. (1, 2) At the same time a centre was developed to study mathematical morphological techniques in Paris school of mines, France. Mathematical morphology can provide solutions to many tasks, where image processing can be applied, such as in remote sensing, optical character recognition, Radar image sequence recognition, medical image processing etc.,

In mathematical morphological operations, Erosion and Dilation are primitive operations (3,4). But there will exist some type of rigidity in mathematical morphological operations. That rigidity is relaxed and the morphological operations are redesigned to overcome some inconveniences, as well as to get some advantages. The primitive operations, Erosion and Dilation, now are called Soft Erosion and Soft Dilation.

The properties of soft mathematical morphological operations are not discussed thoroughly , till now.That gap is tried to be filled in this paper.

II. BACKGROUND

In this section some background concepts are explained.

2.1 Multi scale morphology:

Multi scale morphology has extended its applications to Image Smoothening, Edge Enhancement, Segmentation, Remote Sensing, Radar image analysis, Medical area etc.

In the process of understanding the objective world, the appearance of an object does not depend only on the object itself, but also on the scale that the observer used. It seems that appearance under a specific scale does not give sufficient information about the essence of the percept, we want to understand. If we use a different scale, to examine this percept, it will usually have a different appearance. So, this series of images and its changing pattern over scales reflect the nature of the percept.

The Multi scale morphology is having special applications, like enhancing weak Edges, Decay analysis of wood, critical analysis of (ECG) Cardio imagery (Identification of critical points), Getting results which are helpful for pilots, lunar landing etc.

As the scale is changed the shape of the image also will change. PETROS MARAGOS has concentrated in this area. He explained the applications of MSMM, and back ground mathematics. He explained about application of MSMM in skeletenization also. He extended these concepts to gray scale, also (10). MING – HUA CHEN & PING – GAN YAN also concentrated in this area. They explained (11)

Erosion, Dilation, Open, Close in multi scale environment, with diagrams (results), mathematical analysis, as well as symbolic conventions.

PAUL. T. JACKWAY etc. (16) provided one type of analysis in MSMM. They discussed how to relate the results of one scale with the results at different scale. They have provided this analysis with good examples, using Erosion/Dilation morphological operations. This paper discussed the back ground theory, in one angle, relating to MSMM.

KUN WANG etc. proposed an algorithm, for edge detection in the presence of Gaussian noise & salt – pepper noise in multi scale morphological environment. The experimental results are better than that of conventional algorithms (12). The same authors KUNWANG etc. proposed another algorithm for edge detection (13) which will function better in Gaussian, salt - paper noise environment, in MS morphological approach.

KIM WANG and others discussed an edge detection algorithm, in multi scale environment, which is suitable to apply on brain MRI, in noisy environment. (14).
In addition to above ,some more scientists have done in this area. Only a few research findings are discussed in this paper.

2.2 SOFT MORPHOLOGY

The difference in between morphology and soft morphology is explained very briefly in introduction. A few research findings and applications are discussed in this section.

SHIH, F.Y. etc. discussed (19) soft morphological properties are discussed up to some extent. Some of the properties are stated and idempotency is discussed up to some extent. They discussed about, soft morphology op's in gray scale, using threshold super position theorem. They discussed about implementation of soft morphology op's, using logic gates also. Any way, it discussed soft morphological operations in a few dimensions.

PU, C.C. discussed about (20) implementation of soft morphological op's in gray scale. They integrated super position property and stacking to extend soft morphology from binary scale to gray scale. So this paper provides an opportunity to apply soft morphological operations on all the types of images.(gray scale images also) PAULI KUOSMANEN & JAAKKO ASTOLA (21) also discussed, statistical properties, of soft morphology op's, up to some extent, with connection to stack filters.

GASTERATOS, a discussed (22) a new technique, for the realization of soft morphology op's basing upon majority gate algorithm system architecture, for implementation of soft morphology op's, is also presented. MICHAEL A. ZMUDA (23) proposed an algorithm for implementation of soft morphology ops. Normally voting logic also may be used, across neighborhoods, defined by the S.E.

But, in this algorithm instead of processing all the votes, a few votes may be chosen randomly and the service of FSM also, will be taken, in implementation of this algorithm. It is faster than conventional algorithms. Accuracy: more than 90%.

M. VARDA VOULIA etc. (24) designed algorithms for small detail preservation and impulse noise suppression, using soft morphological op's [soft vector morphology] in color environment and shown better results compared to algorithm designed, based on morphological operators. [Mathematical vector morphology]. A. GASTERATOS, etc. (25) discussed about structuring element decomposition, in soft morphological environment.

In a research paper (26), the authors discussed about properties up to some extent. But, elimination of noise, as well as, detail preservation are opposite characteristics, up to some extent. A strong smoothening filter may not preserve details. But, in soft morphology, a balanced solution maybe obtained, which will preserve details as well as suppress noise, due to flexibility in the definition of soft morphology. It is discussed by KOI VISTO, P. etc. (27).

ZHENG MINGJIE etc. (28) developed directional S.E.'s for speckle noise reduction on SAR images. KOI VISTO, P; etc. (29) concentrated on detail preservation while smoothening. ZHEN JI etc. (30), designed soft morphological filter for reducing periodic noise. These results are compared with other spatial domain as well as frequency domain filters techniques.

ZHEN JI etc. (31), in another research paper, discussed about periodic noise reduction, by soft morphological filters, in another way [another algorithm].MARSHALL, S etc. (32), used soft morphological filters for elimination of disturbance, caused by solar cosmic rays, in the images obtained by astronomy base. [Solar].

So many researchers entered in to edge detection using soft morphological op's. In this way so many researchers have entered and done good research, in soft morphology.

This section explains the importance of soft morphology in image processing. So research in this area is required for future development.

2.3 duals

Applying of dual pairs for smoothening is studied in (17, 18) for better results.

BOUAYNAYA, N; etc. (17) proposed a morphological algorithm using IDEMPOTENCY and DUALTY property for elimination of speckle noise in radar images. [In this paper the importance of duality & idempotency properties are understood]. LEI, T; FAN, Y. Shown (18) elimination of impulse noise by a pair of morphological **dual** operators.

They have shown that, this **dual** pairs provides better results for image smoothing. JIANN–JONE CHEN etc. extended the MSMM to 3D segmentation, using **dual** (MS morphological) concepts (15).

Much work is not done in the study of duality of soft morphological operations. So this property is also taken for the research of the author.

III. DEFINITIONS

In this section , Some definitions are given.

3.1 DUALITY

Dual pairs play a new type of role in various image processing operations like smoothening and edge enhancement etc.

The duality may be defined in the following way, in a very simple manner.

Two operations *, . are duals to each other if $(A*B)=(A^C.B)^C$ or $(A.B)=(A^C*B)^C$

In M.M. $\oplus$ and $\ominus$ are duals. i.e. $(I\oplus S) = (I^C\ominus S)^C$ or $(I\ominus S) = (I^C\oplus S)^C$

In Soft morphology, the duality will exist in a different way because depending upon threshold values, many soft morphological operations will exist.

3.2 SOFT DILATION

In some papers, researchers proposed soft morphology using two sets of structuring elements.
A) The core B) The soft boundary [7,8,9 etc].

But, in some papers [5] they proposed soft morphology, by counting logic. They have done the counting of ones, in the particular sub image, chosen. Then they have applied threshold value, for soft Erosion and soft dilation.

Soft dilation was defined as (5)

$$(I\oplus S^{(m)})\,[x, y] = 1 \text{ If } |I\cap S_{(x,y)}| \geq m$$
$$= 0 \quad \text{otherwise.}$$

Here "m" is threshold value where $1\leq m \leq |S|$. $|S|$ is the cardinality of S.

3.3 SOFT EROSION

Soft Erosion may be defined as

$$(I\ominus S^{(m)})[x, y] = 0 \text{ If } |\bar{I}\cap S_{(x,y)}| \geq m$$
$$= 1 \text{ otherwise.}$$

$\bar{I}$ = inversion of I; m= threshold $\leq|S|$.

IV. DUALS OF SOFT EROSION

The duals of soft erosion was discussed in my paper 38. In this paper the duality of soft dilation is discussed in multi scale environment. According to that research paper ,duals of soft erosion are presented in this section.

4.1. 3/3 structuring element size:

$E(1), E(9)$ are duals.
$E(2), E(8)$ are duals.
$E(3), E(7)$ are duals.
$E(4), E(6)$ are duals.
$E(5), E(5)$ are duals.
$E(6), E4)$ are duals.
$E(7), E(3)$ are duals.
$E(8), E(2)$ are duals.
$E(9), E(1)$ are duals.
$E(5)$ is self dual.

In general $\underline{(E(m))^d = E(10-m)}$ where $m = 1$ to 9. $m = 5$ ... ... self dual.

4.2 5/5 structuring element size:

$E(1), E(25)$ are duals.
$E(2), E(24)$ are duals.
$E(3), E(23)$ are duals.

$E(4), E(22)$ are duals.
$E(5), E(21)$ are duals.
$E(6), E(20)$ are duals.
$E(7), E(19)$ are duals.
$E(8), E(18)$ are duals.
$E(9), E(17)$ are duals.
$E(10), E(16)$ are duals.
$E(11), E(15)$ are duals.
$E(12), E(14)$ are duals.
$E(13)$ is self dual.

In general, $\underline{(E(m))^d = E(26-m)}$ where $m = 1$ to 25
$m = 13$ … … self dual.

4.3. 7/7 structuring element size:

$E(1), E(49)$ are duals.
$E(2), E(48)$ are duals.
$E(3), E(47)$ are duals.
$E(4), E(46)$ are duals.
$E(5), E(45)$ are duals.
$E(6), E(44)$ are duals.
$E(7), E(43)$ are duals.
$E(8), E(42)$ are duals.
$E(9), E(41)$ are duals.
$E(10), E(40)$ are duals.
$E(11), E(39)$ are duals.
$E(12), E(38)$ are duals.
$E(13), E(37)$ are duals.
$E(14), E(36)$ are duals.
$E(15), E(35)$ are duals.
$E(16), E(34)$ are duals.
$E(17), E(33)$ are duals.
$E(18), E(32)$ are duals.
$E(19), E(31)$ are duals.
$E(20), E(30)$ are duals.
$E(21), E(29)$ are duals.
$E(22), E(28)$ are duals.
$E(23), E(27)$ are duals.
$E(24), E(26)$ are duals.
$E(25)$ is self dual.

In general, $\underline{(E(m))^d = E(50-m)}$ where $m = 1$ to 49.
$m = 25$ … … self dual.

4.4. 9/9 structuring element size:

$E(1), E(81)$ are duals.
$E(2), E(80)$ are duals.
$E(3), E(79)$ are duals.
$E(4), E(78)$ are duals.
$E(5), E(77)$ are duals.
$E(6), E(76)$ are duals.
$E(7), E(75)$ are duals.
$E(8), E(74)$ are duals.
$E(9), E(73)$ are duals.
$E(10), E(72)$ are duals.
$E(11), E(71)$ are duals.
$E(12), E(70)$ are duals.
$E(13), E(69)$ are duals.
$E(14), E(68)$ are duals.
$E(15), E(67)$ are duals.
$E(16), E(66)$ are duals.
$E(17), E(65)$ are duals.
$E(18), E(64)$ are duals.
$E(19), E(63)$ are duals.
$E(20), E(62)$ are duals.
$E(21), E(61)$ are duals.
$E(22), E(60)$ are duals.
$E(23), E(59)$ are duals.
$E(24), E(58)$ are duals.
$E(25), E(57)$ are duals.
$E(26), E(56)$ are duals.
$E(27), E(55)$ are duals.
$E(28), E(54)$ are duals.
$E(29), E(53)$ are duals.
$E(30), E(52)$ are duals.
$E(31), E(51)$ are duals.
$E(32), E(50)$ are duals.
$E(33), E(49)$ are duals.
$E(34), E(48)$ are duals.
$E(35), E(47)$ are duals.
$E(36), E(46)$ are duals.
$E(37), E(45)$ are duals.
$E(38), E(44)$ are duals.
$E(39), E(43)$ are duals.
$E(40), E42)$ are duals.
$E(41), E(41)$ is self dual.

In general $\underline{(E(m))^d = E(82-m)}$ where $m = 1$ to 81
$m = 41$ … … self dual.

By means of same logic we can provide duality for $^{\mathbf{11}}\!/_{\mathbf{11}}$, $^{\mathbf{13}}\!/_{\mathbf{13}}$, ….

4.5. Now for general case $^{w}\!/_{w}$ Structuring Element:

Now for general case w / w, where w is structuring element size, we can show that

$E(1), E(w^2)$ are duals.
$E(2), E(w^2-1)$ are duals.
$E(3), E(w^2-2)$ are duals.
$E(4), E(w^2-3)$ are duals.
$E(5), E(w^2-4)$ are duals.
$E(6), E(w^2-5)$ are duals.
$E(7), E(w^2-6)$ are duals.
$E(8), E(w^2-7)$ are duals.
$E(9), E(w^2-8)$ are duals.
$E(10), E(w^2-9)$ are duals.
$E(11), E(w^2-10)$ are duals.
$E(12), E(w^2-11)$ are duals.

… … …
… … …

$\mathrm{E}\left(\frac{w^2+1}{2}-5\right), \mathrm{E}\left(\frac{w^2+1}{2}+5\right)$ are duals.
$\mathrm{E}\left(\frac{w^2+1}{2}-4\right), \mathrm{E}\left(\frac{w^2+1}{2}+4\right)$ are duals.
$\mathrm{E}\left(\frac{w^2+1}{2}-3\right), \mathrm{E}\left(\frac{w^2+1}{2}+3\right)$ are duals.
$\mathrm{E}\left(\frac{w^2+1}{2}-2\right), \mathrm{E}\left(\frac{w^2+1}{2}+2\right)$ are duals.
$\mathrm{E}\left(\frac{w^2+1}{2}-1\right), \mathrm{E}\left(\frac{w^2+1}{2}+1\right)$ are duals.
$\mathrm{E}\left(\frac{w^2+1}{2}\right)$ … … self dual.

In general,

- $(E(m))^d = E(w^2+1-m)$ where $m = 1$ to w^2.

$m = \frac{w^2+1}{2}$ … … self dual.

V. EQUALITY OF SOFT EROSION WITH SOFT DILATION

The equality is discussed in my research paper 37. In that paper ,the equalities of soft erosion and soft dilation are discussed in multi scale environment.

5.1. 3/3 structuring element size:

In this environment the equalities are as given below.

$E(1)=D(9)$ $E(2)=D(8)$
$E(3)=D(7)$ $E(4)=D(6)$
$E(5)=D(5)$ $E(6)=D(4)$
$E(7)=D(3)$ $E(8)=D(2)$
$E(9)=D(1)$

In general, $\underline{E(m)=D(10-m)}$ where m will run from 1 to 9, the threshold value.

$E(1)=D(n^2)$ And $D(1)=E(n^2)$ where n^2 is maximum threshold which is 9 for 3 x 3 structuring element. Here $E(1)$, $D(n^2)$ are pure erosions, and $D(1)$, $E(n^2)$ are pure dilations.

5.2. 5/5 structuring element size

In this environment the equalities are as given below.

$E(1)=D(25)$ $E(2)=D(24)$
$E(3)=D(23)$ $E(4)=D(22)$
$E(5)=D(21)$ $E(6)=D(20)$
$E(7)=D(19)$ $E(8)=D(18)$
$E(9)=D(17)$ $E(10)=D(16)$
$E(11)=D(15)$ $E(12)=D(14)$
$E(13)=D(13)$ $E(14)=D(12)$
$E(15)=D(11)$ $E(16)=D(10)$
$E(17)=D(9)$ $E(18)=D(8)$
$E(19)=D(7)$ $E(20)=D(6)$
$E(21)=D(5)$ $E(22)=D(4)$
$E(23)=D(3)$ $E(24)=D(2)$
$E(25)=D(1)$

In general, $\underline{E(m)=D(26-m)}$ where m will run from 1 to 25, the threshold value.

Note: $E(1)=D(n^2)$ And $D(1)=E(n^2)$ where n^2 is maximum threshold which is 25 for $5/5$ structuring element. Here $E(1)$, $D(n^2)$ are pure erosions, and $D(1)$, $E(n^2)$ are pure dilations.

5.3. 7/7 structuring element size

In this situation also we can have the equalities like above because here also dilation at one threshold will be equal to erosion at the other threshold. Here the threshold value will run from 1 to 49.

$E(1) = D(49)$ $E(2) = D(48)$
$E(3) = D(47)$ $E(4) = D(46)$
$E(5) = D(45)$ $E(6) = D(44)$
……
$E(47) = D(3)$ $E(48) = D(2)$
$E(49) = D(1)$

In general, $\underline{E(m) = D(50 - m)}$ where m = 1 to 49.

Here $E(1)$, $D(49)$ are pure erosions. $D(1)$, $E(49)$ are pure dilations.

5.4. 9/9 structuring element size

In this situation also we can have the equalities like above because here also dilation at one threshold will be equal to erosion at the other threshold. Here the threshold value will run from 1 to 81.

$E(1) = D(81)$ $E(2) = D(80)$
$E(3) = D(79)$ $E(4) = D(78)$
$E(5) = D(77)$ $E(5) = D(77)$
………………………………………………………………………………………………..
$E(79) = D(3)$ $E(80) = D(2)$
$E(81) = D(1)$

In general, $\underline{E(m) = D(82 - m)}$ where m = 1 to 81.

Here $E(1)$, $D(81)$ are pure erosions. $D(1)$, $E(81)$ are pure dilations.

5.5. 11/11 structuring element size

In this situation also we can have the equalities like above because here also dilation at one threshold will be equal to erosion at the other threshold. Here the threshold value will run from 1 to 121.

$E(1) = D(121)$ $E(2) = D(120)$
$E(3) = D(119)$ $E(4) = D(118)$
$E(5) = D(117)$ $E(6) = D(116)$
………
$E(95) = D(27)$ $E(95) = D(27)$
$E(95) = D(27)$
…………………………………………………………………
$E(119) = D(3)$ $E(120) = D(2)$
$E(121) = D(1)$

In general, $\underline{E(m) = D(122 - m)}$ where m = 1 to 121.

Here $E(1)$, $D(121)$ are pure erosions. $D(1)$, $E(121)$ are pure dilations

In the same way we can have equalities in soft erosion and soft dilation in various dimensions of structuring elements, $^{13}/_{13}$, $^{15}/_{15}$, $^{17}/_{17}$, $^{19}/_{19}$, $^{21}/_{21}$, $^{23}/_{23}$,……..

5..6. General case: ……(6)

For structuring element size: $^{w}/_{w}$

➢ $E(m) = D(w^2 + 1 - m)$

VI. MULTIPLE DUALS OF SOFT EROSION

By combining sections 4 and 5, now multiple duals of soft erosion are obtained. They are also given in multi scale environment.

In this discussion the following convention is followed. If dual of "X" is "Y" then it is represented as $X^d = Y$ or $(X)^d = Y$

6..1. $^{3}/_{3}$ Structuring Element

The duals of soft erosion operations are

$(E(1))^d = E(9), D(1)$ $(E(2))^d = E(8), D(2)$
$(E(3))^d = E(7), D(3)$ $(E(4))^d = E(6), D(4)$
$(E(5))^d = E(5), D(5)$ $(E(6))^d = E(4), D(6)$
$(E(7))^d = E(3), D(7)$ $(E(8))^d = E(2), D(8)$
$(E(9))^d = E(1), D(9)$

In general, $\underline{(E(m))^d = E(10 - m), D(m)}$ where m = 1 to 9

6..2. $^{5}/_{5}$ Structuring Element

The duals of soft erosion operations are

$(E(1))^d = E(25), D(1)$ $(E(2))^d = E(24), D(2)$
$(E(3))^d = E(23), D(3)$ $(E(4))^d = E(22), D(4)$
$(E(5))^d = E(21), D(5)$ $(E(6))^d = E(20), D(6)$
$(E(7))^d = E(19), D(7)$ $(E(8))^d = E(18), D(8)$

$(E(9))^d = E(17), D(9)$ $(E(10))^d = E(16), D(10)$
$(E(11))^d = E(15), D(11)$ $(E(12))^d = E(14), D(12)$
$(E(13))^d = E(13), D(13)$ $(E(14))^d = E(12), D(14)$
$(E(15))^d = E(11), D(15)$ $(E(16))^d = E(10), D(16)$
$(E(17))^d = E(9), D(17)$ $(E(18))^d = E(8), D(18)$
$(E(19))^d = E(7), D(19)$ $(E(20))^d = E(6), D(20)$
$(E(21))^d = E(5), D(21)$ $(E(22))^d = E(4), D(22)$
$(E(23))^d = E(3), D(23)$ $(E(24))^d = E(2), D(24)$
$(E(25))^d = E(1), D(25)$

In general, $\underline{(E(m))^d = E(26 - m), D(m)}$ where $m = 1$ to 25

6..3. $^7/_7$ Structuring Element

The duals of soft erosion operations are

$(E(1))^d = E(49), D(1)$ $(E(2))^d = E(48), D(2)$
$(E(3))^d = E(47), D(3)$ $(E(4))^d = E(46), D(4)$
$(E(5))^d = E(45), D(5)$ $(E(6))^d = E(44), D(6)$
$(E(7))^d = E(43), D(7)$ $(E(8))^d = E(42), D(8)$
$(E(9))^d = E(41), D(9)$ $(E(10))^d = E(40), D(10)$

$(E(11))^d = E(39), D(11)$ $(E(12))^d = E(38), D(12)$
$(E(13))^d = E(37), D(13)$ $(E(14))^d = E(36), D(14)$
$(E(15))^d = E(35), D(15)$ $(E(16))^d = E(34), D(16)$
$(E(17))^d = E(33), D(17)$ $(E(18))^d = E(32), D(18)$
$(E(19))^d = E(31), D(19)$ $(E(20))^d = E(30), D(20)$
$(E(21))^d = E(29), D(21)$ $(E(22))^d = E(28), D(22)$
$(E(23))^d = E(27), D(23)$ $\left(E(24)\right)^d = E(26), D(24)$
$\left(E(25)\right)^d = E(25), D(25)$ $\left(E(26)\right)^d = E(24), D(26)$
$\left(E(27)\right)^d = E(23), D(27)$ $(E(28))^d = E(22), D(28)$
$(E(29))^d = E(21), D(29)$ $(E(30))^d = E(20), D(30)$
$(E(31))^d = E(19), D(31)$ $(E(32))^d = E(18), D(32)$
$(E(33))^d = E(17), D(33)$ $(E(34))^d = E(16), D(34)$
$(E(35))^d = E(15), D(35)$ $(E(36))^d = E(14), D(36)$
$(E(37))^d = E(13), D(37)$ $(E(38))^d = E(12), D(38)$
$(E(39))^d = E(11), D(39)$ $(E(40))^d = E(10), D(40)$
$(E(41))^d = E(9), D(41)$ $(E(42))^d = E(8), D(42)$
$(E(43))^d = E(7), D(43)$ $(E(44))^d = E(6), D(44)$
$(E(45))^d = E(5), D(45)$ $(E(46))^d = E(4), D(46)$
$(E(47))^d = E(3), D(47)$ $(E(48))^d = E(2), D(48)$
$(E(49))^d = E(1), D(49)$

In general, $\underline{(E(m))^d = E(50 - m), D(m)}$ where $m = 1$ to 49

6..4. $^9/_9$ Structuring Element

The duals of soft erosion operations are

$(E(1))^d = E(81), D(1)$ $(E(2))^d = E(80), D(2)$
$(E(3))^d = E(79), D(3)$ $(E(4))^d = E(78), D(4)$
$(E(5))^d = E(77), D(5)$ $(E(6))^d = E(76), D(6)$

$(E(7))^d = E(75), D(7)$ $(E(8))^d = E(74), D(8)$
$(E(9))^d = E(73), D(9)$ $(E(10))^d = E(72), D(10)$
$(E(11))^d = E(71), D(11)$ $(E(12))^d = E(70), D(12)$
$(E(13))^d = E(69), D(13)$ $(E(14))^d = E(68), D(14)$
$(E(15))^d = E(67), D(15)$ $(E(16))^d = E(66), D(16)$
$(E(17))^d = E(65), D(17)$ $(E(18))^d = E(64), D(18)$
$(E(19))^d = E(63), D(19)$ $(E(20))^d = E(62), D(20)$
$(E(21))^d = E(61), D(21)$ $(E(22))^d = E(60), D(22)$
$(E(23))^d = E(59), D(23)$ $(E(24))^d = E(58), D(24)$
$\left(E(25)\right)^d = E(57), D(25)$ $\left(E(26)\right)^d = E(56), D(26)$
$(E(27))^d = E(55), D(27)$ $\left(E(28)\right)^d = E(54), D(28)$
$\left(E(29)\right)^d = E(53), D(29)$ $(E(30))^d = E(52), D(30)$
$(E(31))^d = E(51), D(31)$ $(E(32))^d = E(50), D(32)$
$(E(33))^d = E(49), D(33)$ $\left(E(34)\right)^d = E(48), D(34)$
$\left(E(35)\right)^d = E(47), D(35)$ $(E(36))^d = E(46), D(36)$
$\left(E(37)\right)^d = E(45), D(37)$ $\left(E(38)\right)^d = E(44), D(38)$
$(E(39))^d = E(43), D(39)$ $\left(E(40)\right)^d = E(42), D(40)$
$\left(E(41)\right)^d = E(41), D(41)$ $(E(42))^d = E(40), D(42)$
$\left(E(43)\right)^d = E(39), D(43)$ $\left(E(44)\right)^d = E(38), D(44)$
$(E(45))^d = E(37), D(45)$ $\left(E(46)\right)^d = E(36), D(46)$
$\left(E(47)\right)^d = E(35), D(47)$ $(E(48))^d = E(34), D(48)$
$(E(49))^d = E(33), D(49)$ $(E(50))^d = E(32), D(50)$
$(E(51))^d = E(31), D(51)$ $(E(52))^d = E(30), D(52)$

$(E(53))^d = E(29), D(53)$ $(E(54))^d = E(28), D(54)$
$(E(55))^d = E(27), D(55)$ $(E(56))^d = E(26), D(56)$
$(E(57))^d = E(25), D(57)$ $(E(58))^d = E(24), D(58)$
$(E(59))^d = E(23), D(59)$ $(E(60))^d = E(22), D(60)$
$(E(61))^d = E(21), D(61)$ $(E(62))^d = E(20), D(62)$

$(E(63))^d = E(19), D(63)$ $(E(64))^d = E(18), D(64)$
$(E(65))^d = E(17), D(65)$ $(E(66))^d = E(16), D(66)$
$(E(67))^d = E(15), D(67)$ $(E(68))^d = E(14), D(68)$
$(E(69))^d = E(13), D(69)$ $(E(70))^d = E(12), D(70)$
$(E(71))^d = E(11), D(71)$ $(E(72))^d = E(10), D(72)$
$(E(73))^d = E(9), D(73)$ $(E(74))^d = E(8), D(74)$
$(E(75))^d = E(7), D(75)$ $(E(76))^d = E(6), D(76)$
$(E(77))^d = E(5), D(77)$ $(E(78))^d = E(4), D(78)$
$(E(79))^d = E(3), D(79)$ $(E(80))^d = E(2), D(80)$
$(E(81))^d = E(1), D(81)$

In general, $\underline{(E(m))^d = E(82-m), D(m)}$ where $m = 1$ to 81

6..5. w/w Structuring Element

$$(E(1))^d = E(w^2), D(1)$$
$$(E(2))^d = E(w^2-1), D(2)$$
$$(E(3))^d = E(w^2-2), D(3)$$
$$(E(4))^d = E(w^2-3), D(4)$$
$$(E(5))^d = E(w^2-4), D(5)$$
$$(E(6))^d = E(w^2-5), D(6)$$
$$(E(7))^d = E(w^2-6), D(7) \ldots\ldots\ldots$$
$$\ldots\ldots\ldots$$
$$(E(w^2-6))^d = E(7), D(w^2-6)$$
$$(E(w^2-5))^d = E(6), D(w^2-5)$$
$$(E(w^2-4))^d = E(5), D(w^2-4)$$
$$(E(w^2-3))^d = E(4), D(w^2-3)$$
$$(E(w^2-2))^d = E(3), D(w^2-2)$$
$$(E(w^2-1))^d = E(2), D(w^2-1)$$
$$(E(w^2))^d = E(1), D(w^2)$$

In general, $(E(m))^d = E(w^2+1-m), D(m)$

Where $m = 1$ to w^2

VII .CONCLUSION

The dual of soft morphological operations are having wide range of applications. So in this paper the multiple duals of soft erosion in multi scale environment is discussed in detail.

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